

GRAVITY FROM 1600 TO TODAY

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Abstract

The workshop will allow you to explore the development of our understanding of gravity as one of the four fundamental forces of nature. From the realisation that its strength decreases with distance to its properties as a wave.

- The discovery by Kepler of the three laws of planetary motion – 1600
- The use of these by Newton to create his Law of Gravitation – 1687
- The Schiehallion experiment by Maskelyne to measure the density of the Earth – 1774
- Experiments by Cavendish and later, Boys to measure the Gravitational Constant – 1798
- Airy's measurement of the density of the Earth by taking a clock down a coal mine – 1826
- Eddington's observations of the Solar Eclipse of 1919 to verify Einstein's theory – 1919
- The recent measurement of gravitational waves caused by massive objects – 2015.

There will be opportunities to do some simple calculations, to perform a simple pendulum experiment and to complete some diagrams.

Johannes Kepler and Tycho Brahe

Johannes Kepler was a German mathematician, astronomer and astrologer. He is best known for his laws of planetary motion. His work provided one of the foundations for Isaac Newton's theory of universal gravitation.

In 1600, Kepler started to work with Tycho Brahe, a Danish astronomer. Brahe had made accurate measurements of the orbit of Mars. Kepler had a mathematical theory to describe the orbit. Brahe died in 1601 and Kepler was appointed to succeed him as imperial mathematician in the Court of Archduke Ferdinand. He spent the next 11 years refining and testing his theories.

Kepler searched for harmonies in the motion of the heavens. In 1618 he proposed what has become known as his Third Law of Planetary Motion – the square of the periodic times are proportional to the cubes of the mean distances. He also favoured natural geometric shapes hence his interest in the ellipse. (And also – Nova stereometria doliorum vinariorum - New Stereometry of Wine Barrels!)



Figure 1: Johannes Kepler 1571 – 1630

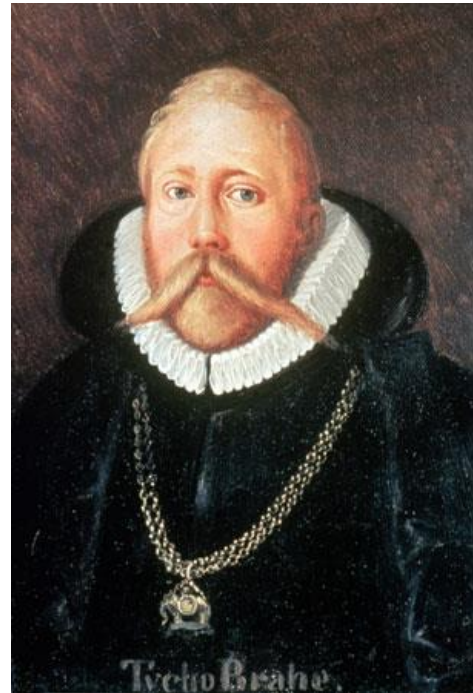


Figure 2: Tycho Brahe 1546 – 1601

Kepler's laws of planetary motion are:

- The planets orbit the Sun in ellipses with the Sun at one focus.
- The line joining the planet and the Sun sweeps out equal areas in equal intervals of time.
- The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

Exercises

Confirm Kepler's Second Law by drawing very fine lines from the Sun to the three dates shown on Comet Halley's orbit. Count the squares in the two triangles to confirm that their areas are equal to each other.

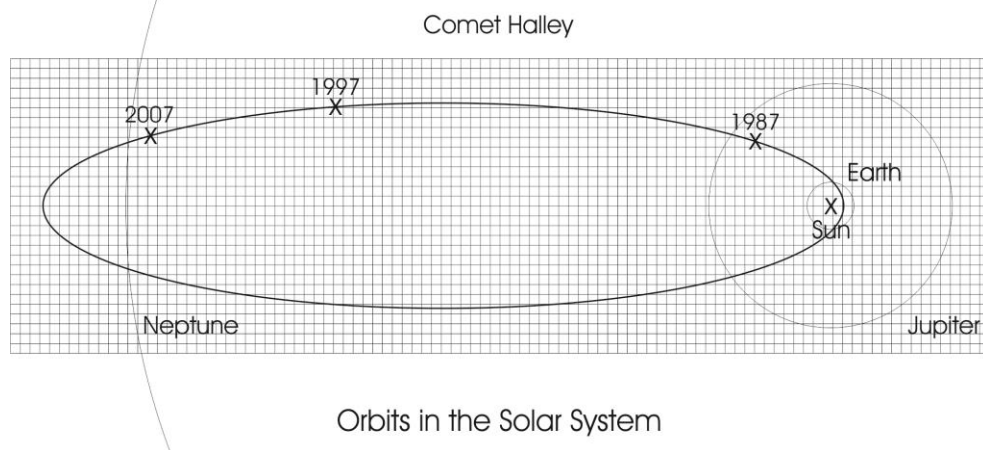


Figure 3: Orbits in the Solar System

Confirm Kepler's Third Law by completing the table below:

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Orbital Period (days)	88.0	224.7	365.2	687.0	4331	10,747	30,589	59,800
Orbital Period Squared								
Distance from Sun (10^6 km)	57.9	108.2	149.6	227.9	778.6	1433.5	2872.5	4495.1
Distance from Sun Cubed								
Period Squared / Distance Cubed								

Table 1: Kepler's Third Law

Newton's Law of Universal Gravitation

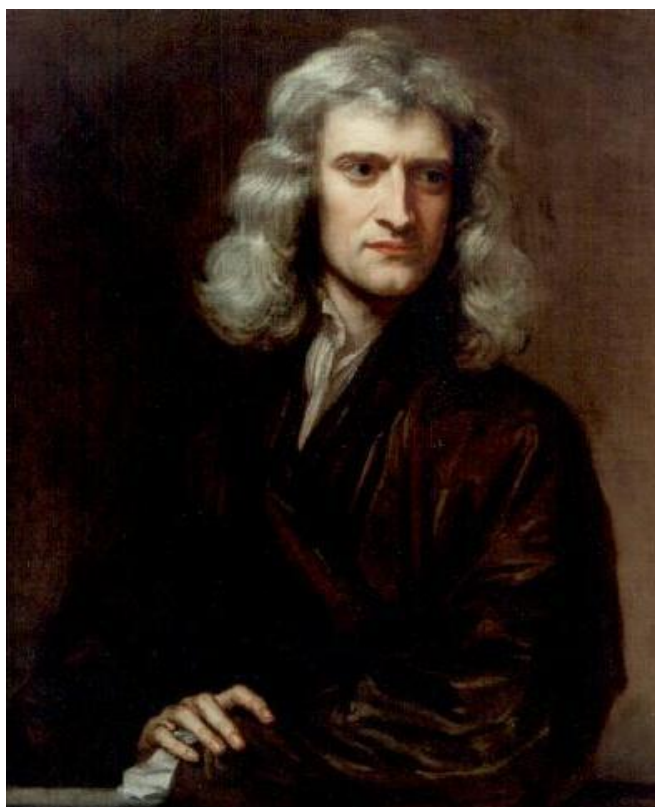


Figure 4: Sir Isaac Newton 1642 – 1727

Newton was able to link Christiaan Huygens' Law of Centrifugal Force with Kepler's Laws of Planetary Motion.

He showed that the force of gravity decreases with the square of the distance between the Sun and the planets. This was a major step forward as both Kepler and Galileo had assumed that the force did not vary with distance, as might be concluded from ground-based experiments.

Newton's theory enjoyed its greatest success when it was used to predict the existence of Neptune. This discovery was based on the motions of Uranus that could not be accounted for by the actions of the other planets.

However, the theory did not fully explain the precession of the perihelion of the orbits of the planets, especially that of planet Mercury, which was detected long after the life of Newton. A solution to this problem had to wait for Einstein.

Nevil Maskelyne

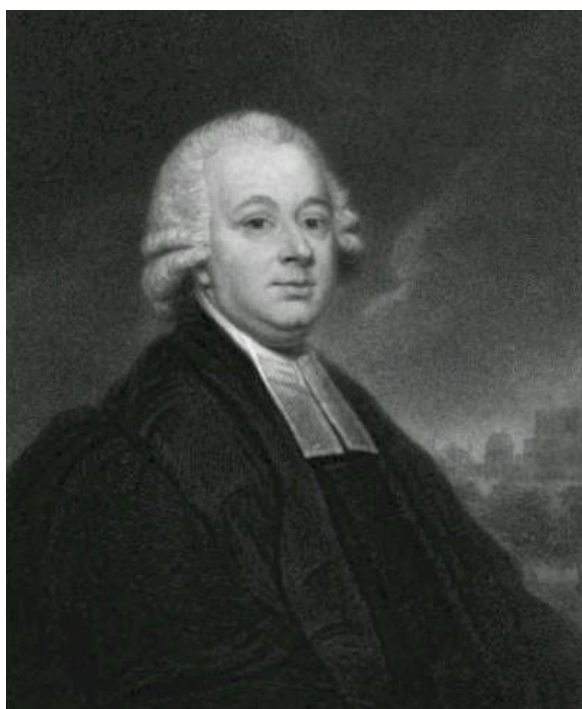


Figure 5: Nevil Maskelyne 1732 – 1811



Figure 6: Schiehallion in Scotland 1083 metres

The Schiehallion Experiment to measure the Density of the Earth (1774)

Nevil Maskelyne, the Astronomer Royal, proposed to measure the horizontal pull of gravity of a large mountain. The Scottish mountain Schiehallion was chosen because it was isolated and fairly symmetrical. It was surveyed carefully so that its volume and centre of gravity could be found. Then very precise measurements of the stars were made from ground level at the side of the mountain. The local vertical is disturbed by the pull of the mountain and so the star positions are slightly shifted.

The astronomical instruments included a 12-inch (30 cm) brass quadrant from Cook's 1769 Transit of Venus expedition, a 10-foot (3.0 m) zenith sector and a regulator (precision pendulum clock) for timing the astronomical observations.

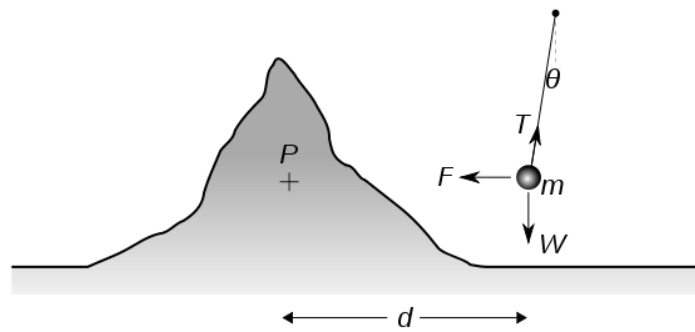


Figure 7: Deflection of a Pendulum

From the South observatory, he was able to take 76 measurements on 34 stars in one direction and then 93 observations on 39 stars in the other. From the North side he then conducted a set of 68 observations on 32 stars and a set of 100 on 37 stars.

Maskelyne showed that the difference between the locally determined zenith for observers North and South of Schiehallion was 54.6 arc seconds. Once the surveying team had provided a difference of 42.94 seconds of latitude between the two stations, he was able to subtract this, and concluded that the sum of the North and South deflections was 11.6 seconds of arc. He calculated that the mean density of the Earth was approximately double that of Schiehallion. Maskelyne had confirmed that Schiehallion exhibited a gravitational attraction. He considered this support for Newton's inverse square law of gravitation.

Demonstration

Water and laser beam reflected to show local horizontal.

Henry Cavendish and Charles Boys



Figure 8: Henry Cavendish 1731 – 1810



Figure 9: Charles Vernon Boys 1855 – 1944

In 1798 Henry Cavendish performed an experiment to measure the density of the Earth that has come to be known as the Cavendish and Boys Experiment. He used a torsion balance built by Englishman and geologist John Michell, who died before he could begin the experiment.

The experimental apparatus consisted of a torsion balance with a pair of 2-inch diameter (0.73 kg) lead spheres suspended from the arm of the balance and two much larger stationary lead balls (158 kg). Cavendish measured the force of gravitational attraction between the spheres. The force involved was very small so he isolated the apparatus in a separate room with external controls and telescopes for making observations.

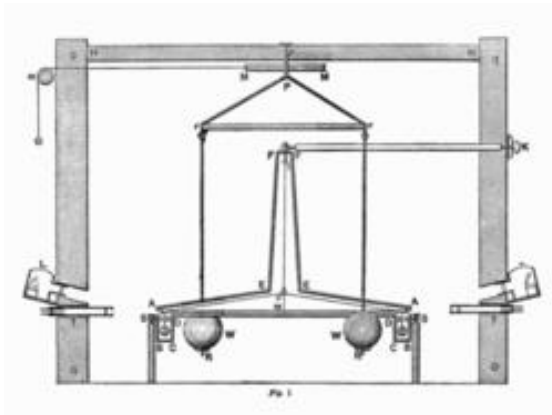


Figure 10: The Torsion Balance – 1.8 m across

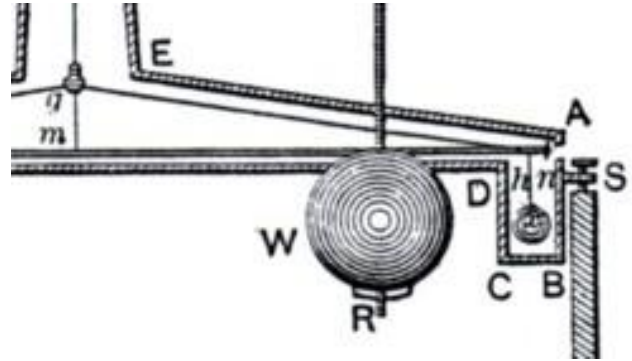


Figure 11: Detail of the two spheres – 0.73 and 158 kg

The deflection stopped when the force of attraction was opposed by the torsion in the supporting wire. The angle was carefully measured and then the long bar was allowed to oscillate freely. Its period was about 20 minutes. From this, the torsion could be deduced and hence the force of attraction.

Cavendish found that the Earth's average density is 5.48 times greater than that of water. This was within 1% of the accepted value.

Over time his results were reformulated to allow the constant in Newton's formula to be calculated. This constant is represented by the capital letter G and is referred to as "Big G ". This distinguishes it from g , the acceleration due to gravity at the surface of the Earth.

This is represented in the illustration below:

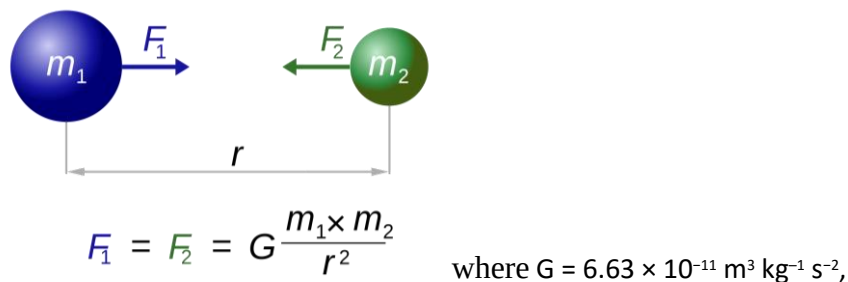


Figure 12: Attraction of two spheres

In 1895 Boys published a measurement of the gravitational constant G that improved upon the accuracy achieved by Cavendish. Boys' method relied on the same theory as Cavendish's, but used two masses suspended at one height and two the other two masses suspended at a different height, to minimize the unwanted interaction between opposite masses.

Calculation and Demonstration

Use the formula on the previous page to calculate the force between you and the Earth. The mass of the Earth is 6.0×10^{24} kg and its radius is 6.4×10^6 m. The answer will be in Newtons of force. If you divide by 9.81 it should give your weight in kilograms.

Demonstration of the torsion balance using magnets to create the forces.

George Biddell Airy



Figure 13: George Biddell Airy 1801 – 1892

The Pit and the Pendulum 1826, 1828 and 1854

George Biddell Airy was professor of Astronomy at the University of Cambridge. In 1826 he proposed an experiment to measure the density of the Earth. His plan was to measure the acceleration due to gravity at the bottom of a deep mineshaft and at the surface above, using pendulums.

When we stand on the surface of the Earth the pull of gravity is due to all the material beneath our feet. If we go below the surface, the pull is due to the smaller sphere that we are standing on. The material above us does not count. So if we go down 100 metres, the material above us pulls upwards and the material in this 100 metre shell that is far away pulls us downwards. The forces cancel out. This has to be, because at the centre of the Earth, the force will be zero. You will be weightless!!

Airy's first attempt, in 1826, was at the Dolcoath copper mine in Cornwall. He set up free-swinging pendulums at the surface and 365 metres below. This required climbing down fifty near-vertical ladders! The periods of these pendulums were measured with clockwork chronometers. The experiment ended abruptly when one of the pendulums was dropped by accident down the mine shaft!!

In 1828, Airy returned and collected more than 127 hours of data. This showed that the lower pendulum swung faster than the upper one. This unexpected result showed that the crust of the Earth had a lower density than the core. This set of experiments ended when a fall of rock caused additional water to enter the mine that the pumps could not remove!!

Some 26 years later, Airy installed his pendulums in the Harton coal mine at South Shields. The mine was 384 metres deep and by this time a mechanical lift was available. A wooden shed was built to house the equipment at the bottom of the mine. This kept the temperature constant and kept the coal-dust away. An electrical circuit was used to measure the coincidences of the pendulums. After three weeks of running, he found that the lower pendulum gained on the upper one by 2.5 seconds per day. After much calculation, Airy published his result that the Earth was 6.565 times more dense than water. This was 20% higher than other methods and was probably due to variations in the density of the rock in the area. However, the method of swinging pendulums to measure the acceleration due to gravity became widely used in geological surveying.

Experiment and Calculation

Measuring the Period Difference of two pendulums made with sand in small plastic pots. Add material to make the periods equal to each other. Then push an empty tube in one pot and remove the displaced sand. The period is not altered. It is the length that matters, not the mass.

Time one pendulum and measure the distance between the top of the string and the centre of the pot. Calculate the acceleration due to gravity using:

$$g = 4 \times \pi^2 \times \text{length} / (\text{period})^2$$

The accepted value is 9.81 m s^{-2} .

Arthur Stanley Eddington



Figure 14: Arthur Stanley Eddington 1882 – 1944



Figure 15: Albert Einstein 1879 – 1955

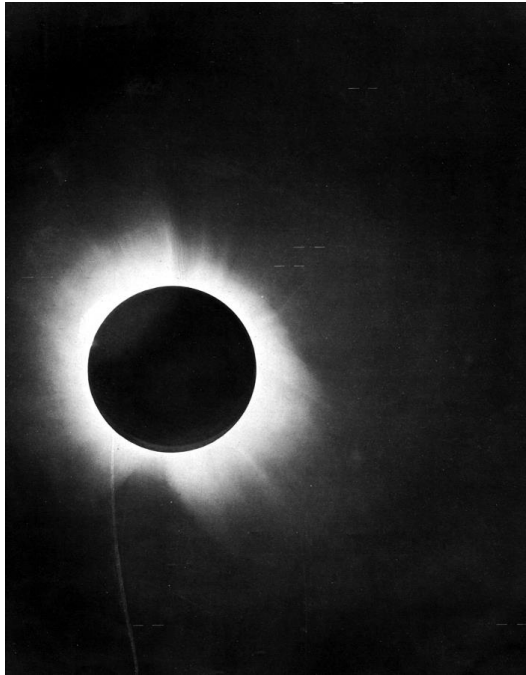


Figure 16: The solar eclipse of 29 May 1919

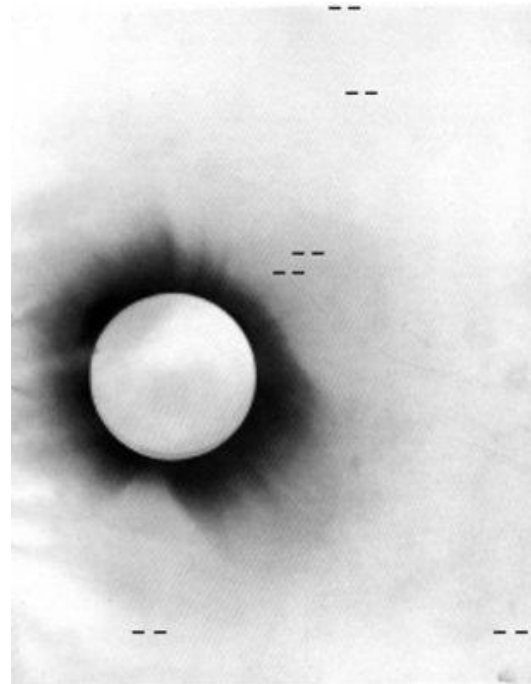


Figure 17: Negative image with markers for the stars

Eddington travelled to the island of Príncipe off the West coast of Africa to watch the Solar Eclipse of 29 May 1919. During the eclipse he took pictures of stars of the Hyades cluster in the region around the Sun. According to the theory of General Relativity, stars whose light rays pass close to the Sun should appear to be shifted slightly because their light has been curved by the Sun's gravitational field.

Eddington had previously observed the Hyades star cluster from Oxford at night time in January and February 1919. He compared the two images and found the deflection to be 1.61 ± 0.30 arcseconds, while Einstein predicted 1.75 arcseconds. This was in good agreement as the very small shift was difficult to measure.

The bending of the rays can be thought of as making a lens. Gravitational lensing has more recently yielded much information about the total mass of galaxy clusters and allowed us to detect very distant galaxies.

Calculation

Measuring an original plate on the next page and consider how small the movement of the image is.

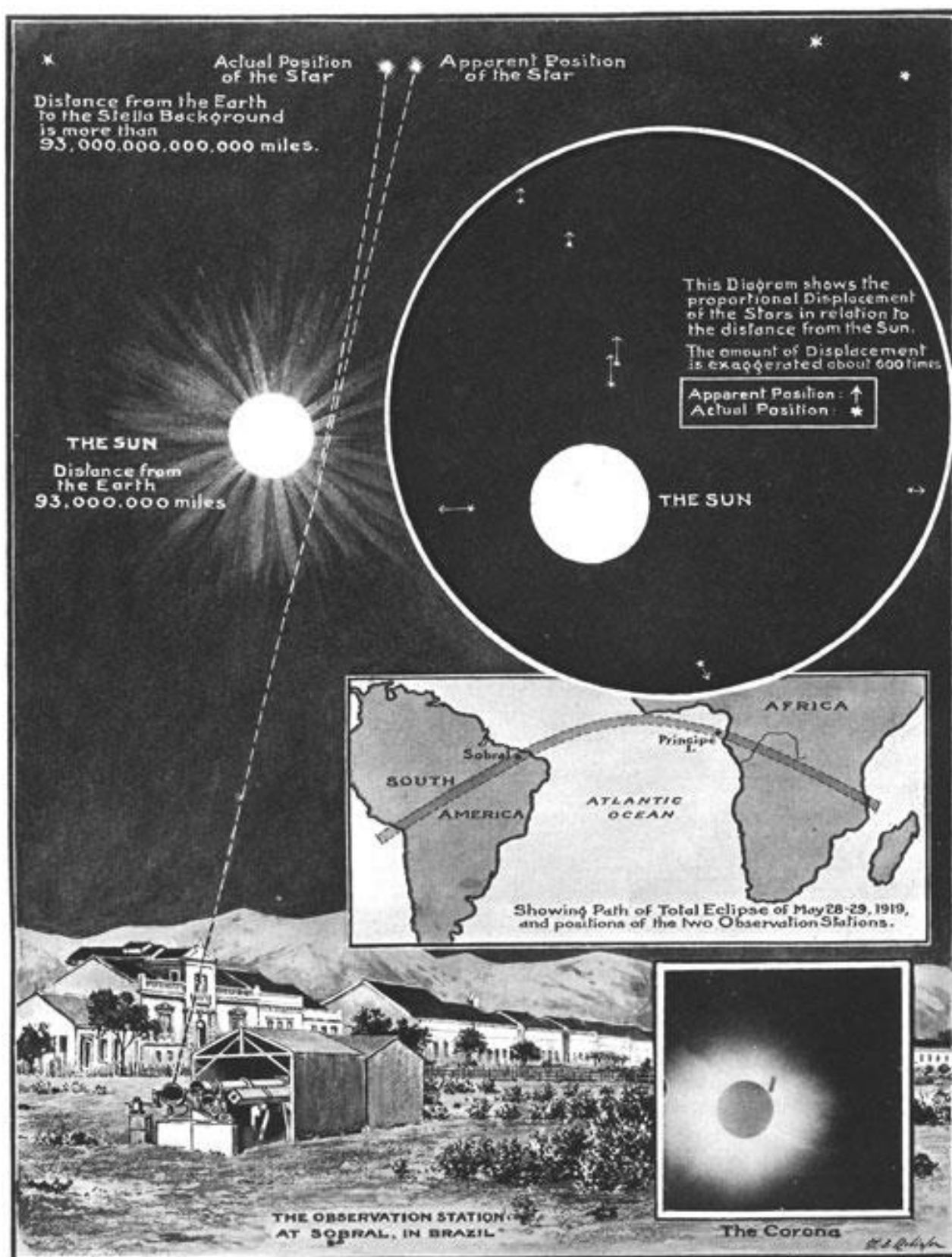


Figure 18: From the Illustrated London News of November 22, 1919

Gravitational Waves 2016

The outstanding observation in astrophysics of the last 100 years happened on 14 September 2015 at 09:51 GMT. A gravitational wave was observed passing two detectors in America. The wave squeezed and stretched the long arms of the detectors shown below. The wave originated from the merger of two black holes about a billion light years away. The discovery was announced on 2 February 2016.

Einstein predicted gravitational waves in his theory of General Relativity, the same theory that predicted the bending of light by massive objects. It only took a few years for Eddington to observe the bending but it took 100 years to observe the waves. There is a good reason why the waves are difficult to observe. It requires a great catastrophe such as the merging or collapsing of stars to make a significant wave and this is quite rare. Then you need a piece of equipment to measure a change in length to one part in 1,000,000,000,000,000,000.



Figure 19: LIGO Hanford Observatory

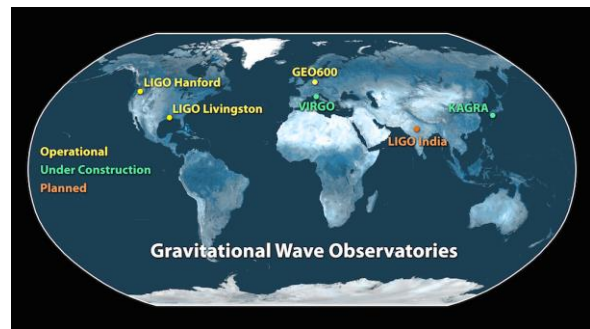


Figure 20: Gravitational-Wave Observatories Across the Globe

The Laser Interferometers of the Gravitational-wave Observatory (LIGO) detectors are located in Livingston, Louisiana and Hanford, Washington, USA.

At each observatory, the L-shaped LIGO interferometer uses laser light split into two beams that travel back and forth down the arms that are 4 km long. The arms are 1.2 metre diameter tubes kept under a near-perfect vacuum. The laser beams are used to monitor the distance between mirrors positioned at the ends of the arms. According to Einstein's theory, the distance between the mirrors will change by a very small amount when a gravitational wave passes through the detector.

Based on the observed signals, LIGO scientists estimate that the black holes for this event were about 29 and 36 times the mass of the Sun, and the event took place 1.3 billion years ago. About 3 times the mass of the Sun was converted into gravitational waves in a fraction of a second – with a peak power output about 50 times that of the whole visible universe. By looking at the time of arrival of the signals – the detector in Livingston recorded the event 6.9 milliseconds before the detector in Hanford – scientists can say that the source was located in the Southern Hemisphere.

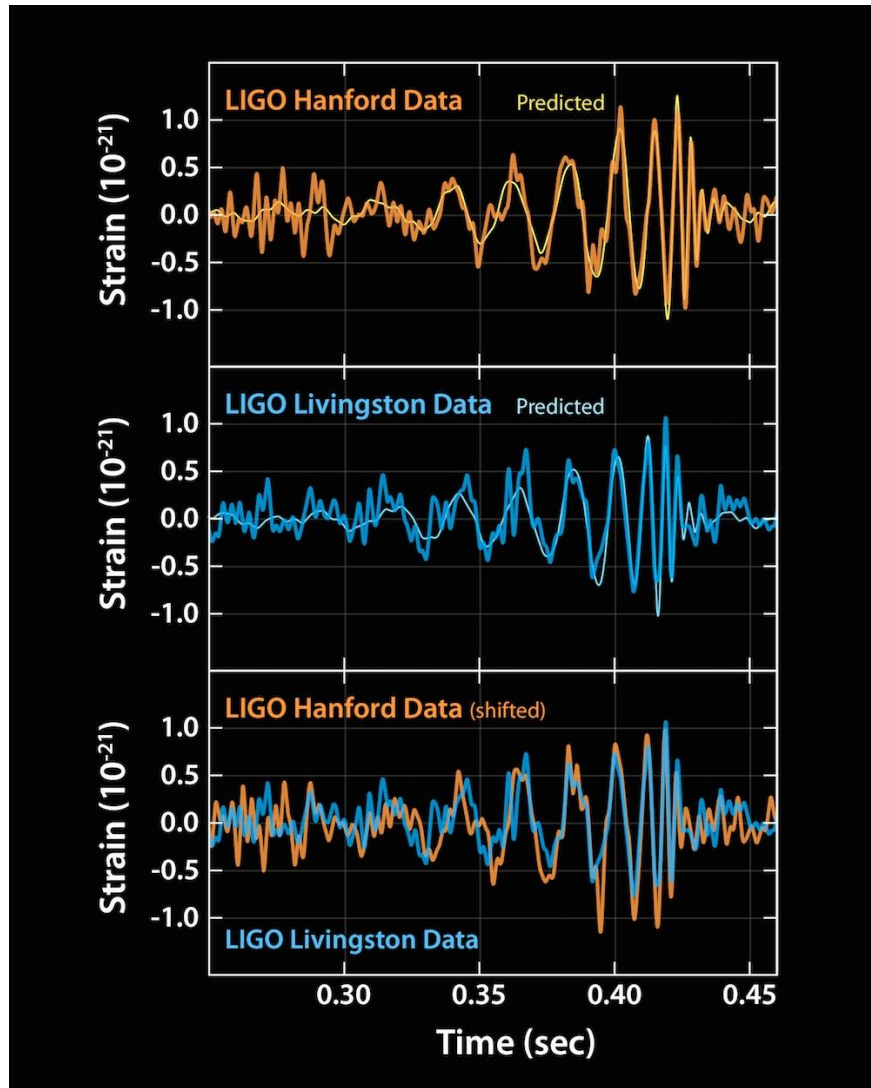


Figure 21: The first observed Gravitational Wave

The top two plots show data received at Livingston and Hanford, along with the predicted shapes for the waveform. These predicted waveforms show what two merging black holes should look like according to the equations of Albert Einstein's general theory of relativity, along with the instrument's ever-present noise. Time is plotted on the x-axis and strain on the y-axis. Strain represents the fractional amount by which distances are distorted. As the plots reveal, the LIGO data very closely matches Einstein's predictions.

The final plot compares data from both detectors. The Hanford data have been inverted for comparison, due to the differences in orientation of the detectors at the two sites. The data were also shifted to correct for the travel time of the gravitational wave signals between Livingston and Hanford (the signal first reached Livingston, and then, traveling at the speed of light, reached Hanford seven thousandths of a second later). As the plot demonstrates, both detectors witnessed the same event, confirming the detection.

Future observations of Gravitational Waves

LIGO is sensitive to only a small part of the Gravitational Wave Spectrum. To detect lower frequencies, it will be necessary to use interferometers in space and to observe the shifts in arrival times of pulsar pulses. The LISA Pathfinder satellite has just been launched and the radio telescope at Parkes in Australia are both actively engaged in this work.

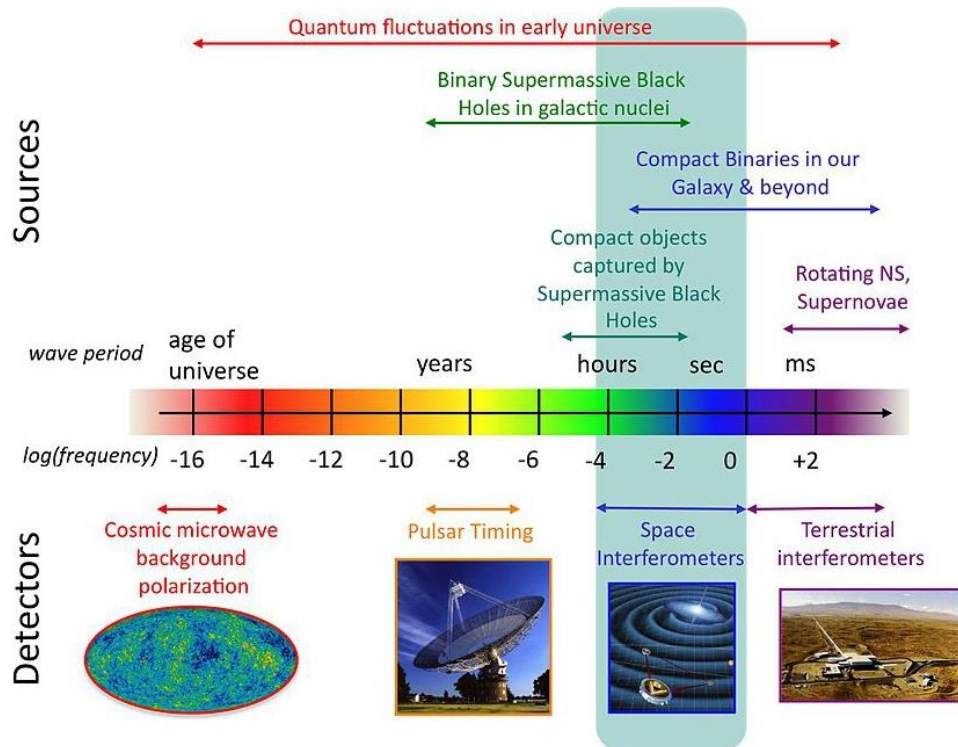


Figure 22: The Gravitational Wave Spectrum

Calculation

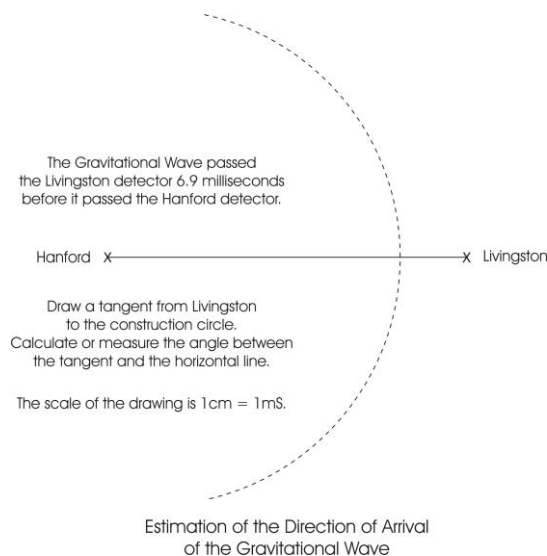


Figure 23: Direction Finding Exercise

Acknowledgement: All images courtesy of Wikipedia except where specifically acknowledged.